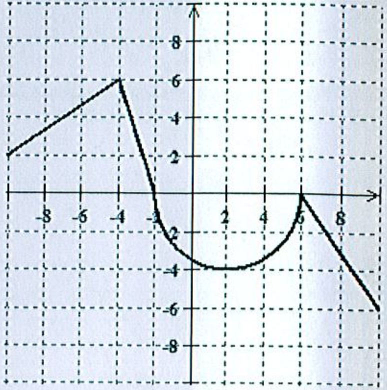
The graph of function f is shown on the right.

SCORE: _____ / 4 PTS

<u>The graph consists of two diagonal lines, an arc of a circle, then an additional diagonal line.</u>



[a] Evaluate $\int_{-10}^{10} f(x)\,dx$.

<u>NOTE: You must show the arithmetic expression that you used to get your answer.</u>

$\frac{1}{2}(6)(2+6) + \frac{1}{2}(2)(6) - \frac{1}{2}\pi(4)^2 - \frac{1}{2}(4)(6)$

$= 24 + 6 - 8\pi - 12$

$= 18 - 8\pi$

$\frac{1}{2}$ POINT EACH EXCEPT AS NOTED

[b] Evaluate $\int_{2}^{-10} f(x)\,dx$.

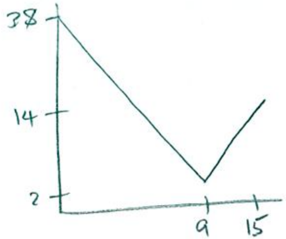
$= -\int_{-10}^{2} f(x)\,dx = -\left[24 + 6 - \frac{1}{4}\pi(4)^2\right] = 4\pi - 30$

1

A person's velocity (in meters per second) at time t (in seconds) is given by $v(t) = \begin{cases} 38 - 4t, & 0 \le t \le 9 \\ 2t - 16, & 9 \le t \le 15 \end{cases}$.

SCORE: ____ / 5 PTS

[a] Find the exact distance the person travelled from time $t = 0$ seconds to $t = 15$ seconds.
NOTE: You must show the arithmetic expression that you used to get your answer.

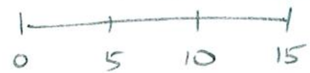


$\underbrace{\tfrac{1}{2}(9)(38+2)}_{1} + \underbrace{\tfrac{1}{2}(6)(2+14)}_{1}$

$= 180 + 48 = \underbrace{228}_{\frac{1}{2}}\text{ m}$

[b] Estimate the distance the person travelled from time $t = 0$ seconds to $t = 15$ seconds using three subintervals and left endpoints.
NOTE: You must show the arithmetic expression that you used to get your answer.

$\Delta x = \dfrac{15-0}{3} = 5$



$f(0)\Delta x + f(5)\Delta x + f(10)\Delta x = \underbrace{38}_{\frac{1}{2}} \cdot 5 + \underbrace{18}_{\frac{1}{2}} \cdot 5 + \underbrace{4}_{\frac{1}{2}} \cdot 5 = \underbrace{300}_{1}\text{ m}$

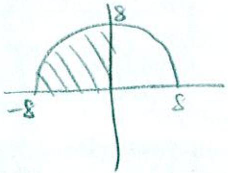
Evaluate $\int_{-8}^{0}(2\sqrt{64-x^2}-|x+6|)\,dx$ using the properties of definite integrals and interpreting in terms of area. **SCORE: _____ / 5 PTS**

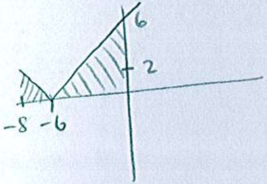
NOTE: You must show the proper use of the properties of the definite integral, NOT just the arithmetic.

2

$$\underbrace{2\int_{-8}^{0}\sqrt{64-x^2}\,dx-\int_{-8}^{0}|x+6|\,dx}=\underbrace{2\cdot\frac{1}{4}\pi(8)^2}_{1}-\Big[\underbrace{\frac{1}{2}\cdot2\cdot2+\frac{1}{2}\cdot6\cdot6}_{1}\Big]$$

$$=\underbrace{32\pi-20}_{1}$$





Using the limit definition of the definite integral, and right endpoints, find $\int_{-3}^{1} (3x^2 + 9x)\,dx$.

SCORE: _____ / 10 PTS

NOTE: Solutions using any other method will earn 0 points.

$$\Delta x = \frac{1 - -3}{n} = \frac{4}{n}$$

$$1 \quad \underbrace{\lim_{n\to\infty} \sum_{i=1}^{n}}_{} \underbrace{\left(3\left(-3+\frac{4i}{n}\right)^2 + 9\left(-3+\frac{4i}{n}\right)\right)}_{2} \underbrace{\frac{4}{n}}_{} \quad 1$$

$$= \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} \left(3\left(9 - \frac{24}{n}i + \frac{16}{n^2}i^2\right) - 27 + \frac{36}{n}i\right)$$

$$= \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} \underbrace{\left(-\frac{36}{n}i + \frac{48}{n^2}i^2\right)}_{} \quad 2$$

$$= \lim_{n\to\infty} \frac{4}{n} \left(-\frac{36}{n}\sum_{i=1}^{n} i + \frac{48}{n^2}\sum_{i=1}^{n} i^2\right)$$

$$= \lim_{n\to\infty} \frac{4}{\cancel{n}} \left(-\frac{\overset{18}{\cancel{36}}}{n} \underbrace{\frac{\cancel{n}(n+1)}{\cancel{2}}}_{1} + \frac{\overset{8}{\cancel{48}}}{n^2} \underbrace{\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}}}_{1}\right)$$

$$= 4(-18+16)$$

$$= \underbrace{-8}_{} \quad 1$$

+ 1 POINT IF YOU WROTE $\lim_{n\to\infty}$ ON EVERY LINE THAT CONTAINS "n"